MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 1 - 2017

Differentiation of Exponential and Trigonometric Functions

Resource Free

Name: SOLUTIONS **Teacher:** ____________

Marks: 18

Time Allowed: 20 minutes

Instructions: You are NOT allowed any Calculators or notes.

You will be supplied with a formula sheet.

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1. Find $\frac{dy}{dx}$ *for*

a) $y = \dfrac{16e^{x}}{4e^{5x}}$

$y = 4e^{-4x} \Rightarrow y' = -16e^{-4x} = \dfrac{-16}{e^{4x}}$ ✓✓✓

$\left[\dfrac{-256e^{6x}}{(4e^{5x})^2}\right]$

b) $y = 2\sin(e^{2x})$

$y' = 2\cos(e^{2x}).\,2e^{2x}$

$= 4e^{2x}.\cos(e^{2x})$

✓✓ ✓

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c) $y = 3x^2e^{2x}$ [simplify your answer]

$y' = 3x^2 \cdot 2e^{2x} + 6xe^{2x}$ ✓✓

$= 6x^2e^{2x} + 6xe^{2x}$

$= 6xe^{2x}(x+1)$ ✓

d) $y = 3\pi \tan(1+e)^2$

$= 0$ ✓✓✓

[3,3,3,3 = 12 Marks]

2. Find the equation of the tangent to the curve defined by $h = (e^{2t})(e^t + 1)^2$ at the point (0,4).

$h = e^{2t}(e^{2t} + 2e^t + 1)$

$= e^{4t} + 2e^{3t} + e^{2t}$

$\Rightarrow h' = 4e^{4t} + 6e^{3t} + 2e^{2t}$

When $t = 0$

$h' = 4 + 6 + 2$

$= 12$ ✓✓

Now

$y = mx + c$

$4 = 12(0) + c$

$\Rightarrow c = 4$ ✓✓

Hence equation of tangent is:

$y = 12x + 4$ ✓✓

[6 Marks]

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MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 1 - 2017
Differentiation of Exponential and Trigonometric Functions
Resource Rich

Name: SOLUTIONS **Teacher:** ______________

Marks: 26

Time Allowed: 25 minutes

Instructions: You are allowed a ClassPad and 1 page of notes (both sides).

You will be supplied with a formula sheet.

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1) It is known that the amount of a dangerous 'recreational drug' (in mg) left unabsorbed in the bloodstream after t hours is given by

$$U = 100e^{-0.05t}$$

a) Show that the rate of change of U with respect to time is proportional to the amount of the drug remaining.

$$\frac{dU}{dt} = -0.05 \times 100e^{-0.05t} \checkmark$$

$$= -0.05U \checkmark$$

Hence $\frac{dU}{dt}$ is proportional to U ✓

b) Find the time taken for 90% of the initial amount of the drug to be absorbed by the bloodstream. Give your answer to the nearest hour.

$$10 = 100e^{-0.05t} \checkmark$$

Solving for $t \approx 46.05$ hours

Hence 46 hours ✓

c) Find an expression that describes the amount of the drug absorbed by the bloodstream after t hours.

Amount absorbed $= 100 - 100e^{-0.05t}$ ✓

[3,2,1 = 6 Marks]

2) a) The normal to a given curve at a point is defined as the perpendicular to the tangent at that point. Find the equation of the normal to the curve $y = \dfrac{e^x}{2-x}$ at the point where $x = 1$.

$$\frac{dy}{dx} = \frac{(2-x)e^x + e^x}{(2-x)^2} \checkmark$$

When $x = 1$ $y = e$ $\frac{dy}{dx} = 2e$ ✓

Gradient of perpendicular $= -\frac{1}{2e}$ ✓

Using $(y - y_1) = \cancel{m}(x - x_1)$

$\Rightarrow y - e = -\frac{1}{2e}(x-1)$

$\therefore y = -\frac{x}{2e} + e + \frac{1}{2e}$ ✓

b) $y = x + 1$ is a tangent to the curve $y = ax + b \sin x$ at the point $(\frac{\pi}{2}, 1 + \frac{\pi}{2})$. Find a and b.

$y = ax + b\sin x$

When $x = \frac{\pi}{2}$, $y = 1 + \frac{\pi}{2}$

$\Rightarrow 1 + \frac{\pi}{2} = a\left(\frac{\pi}{2}\right) + b$ ✓

$\Rightarrow b = 1 + \frac{\pi}{2} - \frac{a\pi}{2}$ $\Rightarrow y = ax + \left(1 + \frac{\pi}{2} - \frac{a\pi}{2}\right)\sin x$

$\frac{dy}{dx} = a + \left(1 + \frac{\pi}{2} - \frac{a\pi}{2}\right)\cos x$ ✓

Tangent at $\left(\frac{\pi}{2}, 1 + \frac{\pi}{2}\right)$ is given as $y = x + 1$

When $x = \frac{\pi}{2}$, $\cos x = 0$ hence

$\frac{dy}{dx} = 1$ hence $a = 1$ ✓ and $b = 1$ ✓

[4,4 = 8 Marks]

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3) Fishermen monitored the growth of the population of sardines in a particular location over a 30 year period from 1985 when the population was estimated to be 2 000 000 . They found that the population was continuously growing with the instantaneous rate of increase in the population per year $\frac{dP}{dt}$, always close to $\frac{P}{20}$.

a) Estimate the population of sardines at the end of the 30 year period.

Note $\frac{1}{20} = 0.05$

hence $\frac{dP}{dt} = 0.05P$

$\Rightarrow \underline{P = P_0 e^{0.05t}}$ ✓

When $t = 30$

$P = 2\,000\,000e^{0.05(30)}$

$\approx \underline{8\,960\,000}$ ✓✓

b) If this pattern of growth continues estimate the population of sardines in 2040.

In 2040, $t = 55$ ✓

When $t = 55$

$P = 2\,000\,000\,e^{0.05(55)}$

$\approx 31\,000\,000$ ✓✓

[3,3 = 6 marks]

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4) The displacement, x cm, of a particle from a fixed point O, t seconds after it is released is modelled by the equation $x = -5\cos\frac{\pi t}{4}$. Use a calculus method to determine:

a) The velocity of the particle after 2 seconds,

$$\frac{dx}{dt} = \frac{5\pi}{4} \sin \frac{\pi t}{4} \checkmark$$

At $t=2$

$$\frac{dx}{dt} = \frac{5\pi}{4} \sin\left(\frac{\pi}{2}\right)$$

$$= \frac{5\pi}{4} \text{ cms}^{-1} \checkmark$$

b) When during the interval $0 \le t \le 8$, the particle travels with a speed of 1 cms^{-1} .

$$\frac{dx}{dt} = \pm 1$$

$$\frac{5\pi}{4} \sin\left(\frac{\pi t}{4}\right) = 1$$

$$\Rightarrow t = 3.67, 0.33 \checkmark\checkmark$$

$$\frac{5\pi}{4} \sin\left(\frac{\pi t}{4}\right) = -1$$

$$\Rightarrow t = 4.33, 7.67 \checkmark\checkmark$$

Hence 4 times:

$$t = 3.67, 0.33, 4.33, 7.67$$

[~~5~~ 6 marks]

End of Test

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